

# SINUSOIDAL FUNCTIONS

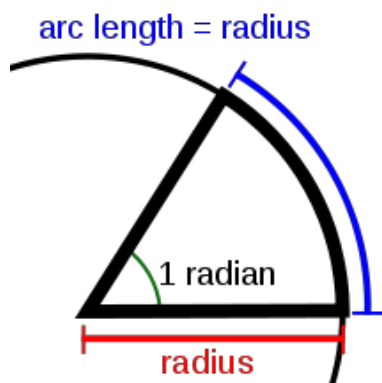
Study Guide | Marko, Brad, Janey, Andy, Jeana, Manas

## MAIN IDEAS

1. The relationship between degree and radian measure is the length of an arc on the unit circle subtended a central angle.
2. Sinusoidal graphs can be graphed using a period with degrees or radians.
3. The graphs of cosine, sine, and tangent functions are periodic because of the motion of an object around circle.
4. Sine, cosine, and tangent graphs all have a basic graph shape that they follow.
5. Equations can involve trigonometric functions.
6. Sinusoidal functions can be used to model real-world applications.
7. A sinusoidal function is a function defined by a formula of the form:  $f(x)=A\cos(ax+b)+B$  or  $f(x)=A\sin(ax+b)+B$ , where  $A, B, a$ , and  $b$  are real numbers.

### 1. The relationship between degree and radian measure is the length of an arc on the unit circle subtended by a central angle.

A central angle is an angle formed from two points on a circle and the origin point. The part of the circle that is between the two sides of the angle is an arc. There is a direct relationship between the length of the arc and the measure of the central angle that defines the arc. A radian is another unit of measure for the central angle. The radian is calculated as the arc length/the radius. 1 radian is the measure of an angle that creates an arc with the same length as the radius of the circle.



### 2. Sinusoidal graphs can use a period of either degrees or radians.

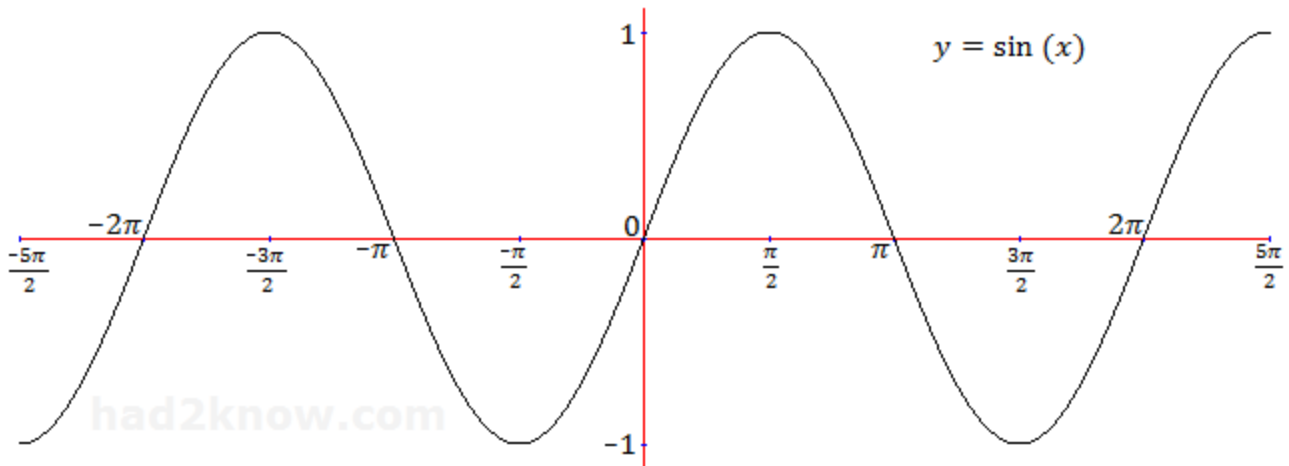
Example: a period of 360 degrees =  $2\pi$  radians. To change between degrees and radians on your calculator, go Mode -> Third Row --> Radian/Degree.

### 3. The graphs of cosine, sine, and tangent functions are periodic because of the motion of an object around a circle.

A unit circle has a diameter of  $2\pi$ , thus the period for a standard cosine, sine, or tangent function is  $2\pi$ .

#### 4. Sine, cosine, and tangent graphs all have a basic graph shape that they follow.

The function  $y = \sin x$  has the following graph:

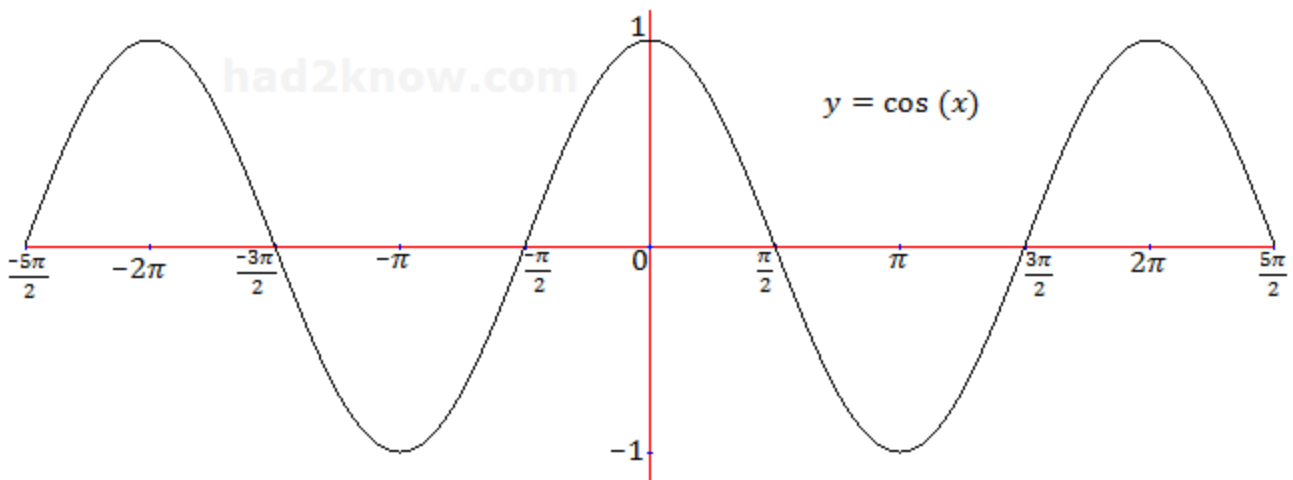


However, this graph can be transformed by stretching, compressing, or shifting horizontally and vertically. In a typical sine graph, the format is  $H(x) = A \sin(ax + b) + B$  where  $A$  determines how much the graph is compressed vertically,  $a$  determines how much the graph is compressed horizontally,  $b$  is the amount shifted horizontally, and  $B$  is the amount shifted vertically.

For example, if  $A = 1/2$ , then the graph is compressed vertically by 2. If  $a = 1/2$ , then the graph is stretched horizontally by 2. If  $b = 1/2$ , then the graph is shifted horizontally to the left (if  $b$  is negative then it's to the right and if  $b$  is positive then it's to the left). If  $B = 1/2$  then the graph is shifted vertically up (if  $B$  is positive then it shifts up and if it's negative then it shifts down).

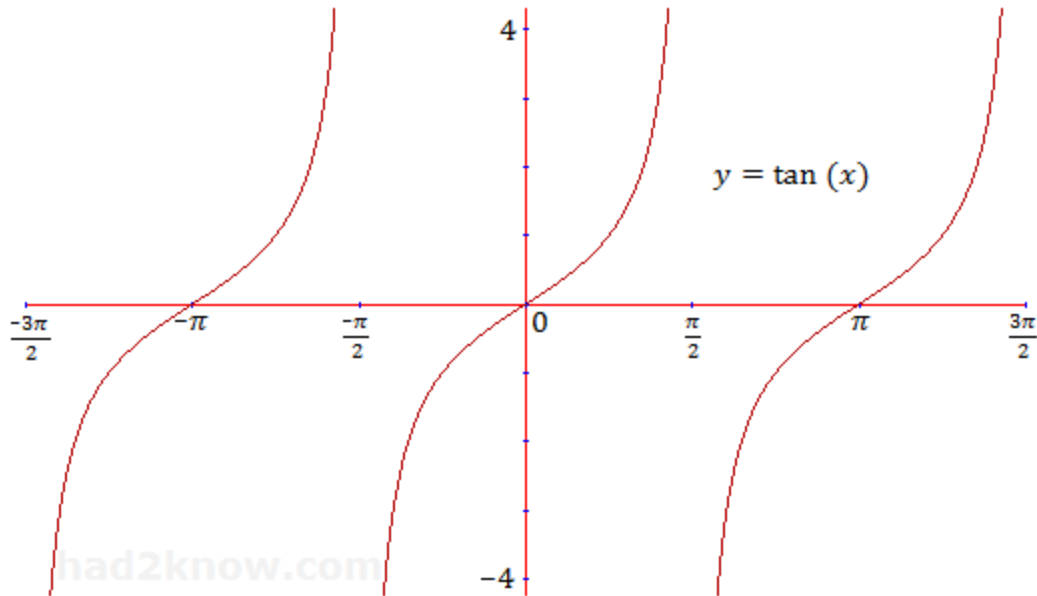
**\*Note:** The periodicity is controlled by the parameter  $a$ .

The function  $y = \cos x$  has the following graph:



Similar to the sine function, the cosine function has the form  $H(t) = A \cos(at + b) + B$  with the parameters having the same meaning as the sine function.

The function  $y = \tan x$  has the following graph:



Similar to the sine function and the cosine function, the tangent function has the form  $H(t) = A \tan(at+b) + B$  with the parameters having the same meaning as the sine and cosine function.

5. Equations can involve trigonometric functions. See example question 1.
6. Sinusoidal functions can be used to model real-world applications. See example question 4.
7. A sinusoidal function is a function defined by a formula of the form:  $f(x) = A \cos(ax+b) + B$  or  $f(x) = A \sin(ax+b) + B$ , where  $A, B, a$ , and  $b$  are real numbers.

## VOCABULARY

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**Amplitude:** the distance from its average value to the maximum or the minimum value

**Arc:** the part of circle that is between the two sides of an angle

**Central Angle:** an angle at the center of the circle

**Phase Shift:** the amount of horizontal translation required to obtain a sinusoidal graph  $f(x)$  from the graph of  $A\cos(ax)$  or  $A\sin(ax)$

**Pythagorean Identity:**  $\cos^2x + \sin^2x = 1$

**Radian:** a measure of cosine or sine of the arc where  $2\pi$  radians is equal to the circumference of the circle.

**Secant Line:** Let  $f$  be a function and suppose  $A$  and  $B$  are distinct points on the graph of  $y = f(x)$ . A line secant to the graph of  $y=f(x)$  is the line passing through  $A$  and  $B$ .

**Sinusoidal Function:** function defined by the form  $f(x)= A \cos(ax+b) + B$  or  $f(x)= A \sin (ax+b) + B$

**Turning Point:** an input  $x$  where the function changes from increasing to decreasing or from decreasing to increasing.

**Vertical Displacement:** the average of the maximum and minimum values of a function.

## PRACTICE PROBLEMS

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1.) Solve the following

$$\sin^2(x) - \sin(x) = 2 \text{ on } 0^\circ < x < 360^\circ$$

2.) a.) Sketch the following two functions on the same axes.

$$d(x)=\cos^2x-\sin^2x$$

$$e(x)=\cos^4x-\sin^4x$$

b.) Explain how the functions  $d(x)$  and  $e(x)$  relate.

3.) Which part of a sinusoidal function can be used to determine the amplitude of the function?

$$f(x) = a \sin x$$

4.) Suppose  $f(t)$  describes the height of a person on the Ferris wheel after some time ( $t$ ).

Based on what you know about how cosine and sine functions are defined using the unit circle, explain why  $f(t)$  must be a sinusoidal function.

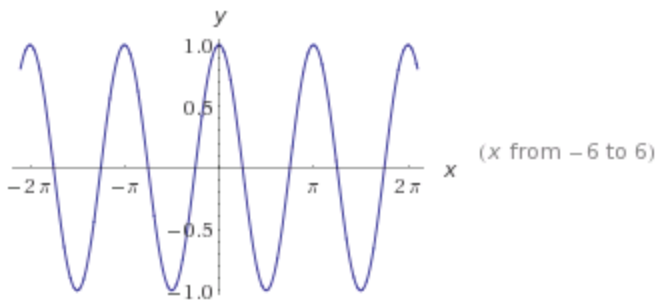
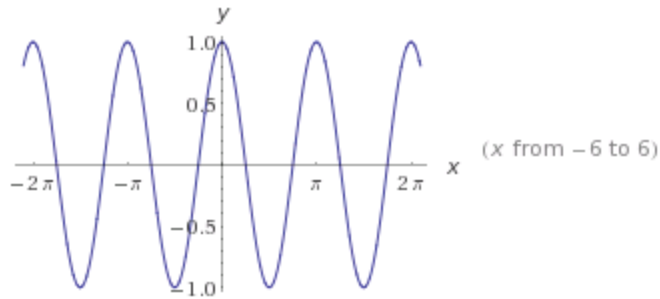
5.) Given triangle ABC with  $CA=7$ ,  $\angle BCA=105^\circ$ , and  $\angle CBA=35^\circ$ , find the length of AB.

6.) Given triangle ABC with  $AB=14$ ,  $BC=10$ , and  $\angle ABC=44^\circ$ , find the length of AC.

## ANSWERS

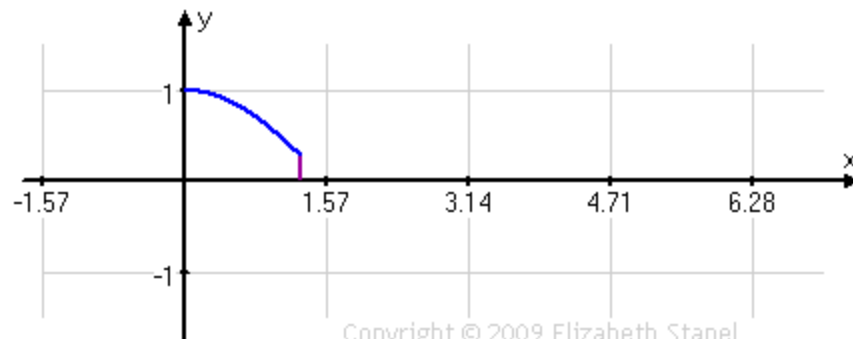
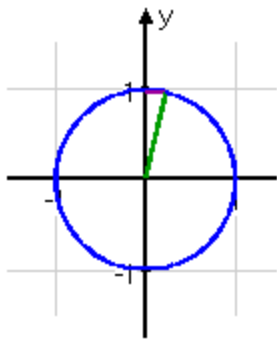
1.)  $\sin(x) = -1$

2.) The graphs are identical



3.) **a** defines the amplitude: if **a** is 3, the amplitude (absolute value of the (maximum-minimum)/2)

4.) As the individual goes around the ferris wheel, the **Y** value constantly fluctuates and peaks and dips as the **t** value (time) increases, creating a sinusoidal graph.



5.) Using law of sines,  $a/\sin(A) = b/\sin(B) = c/\sin(C)$

$$7/\sin(35) = c/\sin(105)$$

$$7 \cdot \sin(105) / \sin(35) = c$$

$$11.788 = c = AB$$

6.) Using the law of cosines,  $b^2 = a^2 + c^2 - 2ac \cdot \cos(44)$

$$x^2 = 14^2 + 10^2 - (2 \cdot 14 \cdot 10) \cdot \cos(44)$$

$$x^2 = 296 - 280 \cdot \cos(44)$$

$$x^2 = 94.585$$

$$x = \sqrt{94.585}$$

$$x = 9.725 = AC$$