Complex Numbers

Essential ideas:

1. **Complex numbers can be represented through graphing.** Complex graph uses the value of *i* in an expression to show the y value. for example, the expression x+i would represent the point (1,1) on the coordinate plane. A vector is an arrow representing the run and the rise of a complex number. many ideas such as conjugate, magnitude and argument can be shown through graphing the complex number on a coordinate plane.

2. **The arithmetic operations of complex numbers are similar to the ones of the real number system.** In complex number arithmetic, you can add and subtract like terms; for example: in addition (a+bi) + (c+di) = (a+c) + (b+d)i and in subtraction:(a+bi) - (c+di) = (a-c) + (b-d)i. In multiplication, complex number arithmetic is just regular multiplying as if you were multiplying roots or polynomials. For example in multiplication, (a+bi)(c+di) = ac + (ad + bc)i - bd. Dividing is very simple just like dividing two polynomials as well. In conclusion, the arithmetic operations on complex numbers work the same as on any polynomials. An important fact to remember is that i^2 = -1, which helps when simplifying an expression in the end.

3. **Complex numbers are identified by a series of properties, including conjugate, magnitude and argument.** For any complex number (z), which can be written as x+yi, there is a conjugate.This is the number written as x-yi, which contains the same x, which is real, but the opposite y, which is imaginary. On the complex plane, the conjugate of any point is its reflection over the x-axis. On this same plane, the magnitude and argument can be measured. The magnitude is a complex number's distance from 0+0i on the plane. The argument is the standard position angle that has x+yi on its terminal side.

4. ***Fundamental Theorem of Algebra* and *Complex Conjugates Theorem* can be applied to find how many zeros polynomials may potentially have, including complex zeros.** The *Fundamental Theorem of Algebra* (for polynomials in the complex number system) explains that a polynomial with degree *n* will have exactly *n* zeros. Repeated zeros are counted separately (ex. The function $f(x) = (x-2i)^2$ has a degree of 2. The function can be also written as f(x)=(x-2*i*)(x-2*i*), the function has two roots (or zeros), "2*i*" and another "2*i*"). The *Complex Conjugates Theorem* explains that for any polynomial with real coefficients and has *a +bi* as a zero, must also have the conjugate *a* - *bi* as a zero. So, any complex non-real zeros will occur in pairs with its conjugate.

5. **Complex numbers can be zeros of quadratic and polynomial functions.** The first step in completely factoring a polynomial is to find factors with all real coefficients, then these can be further factored down using the quadratic formula which is discussed later in the paragraph. It is important to note that the greatest number x is raised to is the number of factors the polynomial has. It is also important to note that if all the coefficients of the polynomial are real numbers,

there must be an even number of complex number factors. A complex number can only be turned into a polynomial with real number coefficients if multiplied by another complex number.

EX: If you know(5-i) is a zero, then x-(5-i) is a factor so (x-5+i) is a factor. However, the only polynomial/factor that one can multiply with (x-5+i) to eliminate the presence of i is (x-5-i).

EX:(x-5+i)*(x-5-i)= x^2 -5x-ix-5x+25+5i+ix-5i-i^2=x^2-10x+25-i^2=x^2-10x+26

Sample Problem: Find the lowest degree polynomial with all real number coefficients with zeros: 4+7i, 3+2i

Answer: x^4-14x^3+102x^2-302x-715

In problems from previous units there were some quadratics that could not be further factored. This is not the case if you are working in the imaginary number system. Each quadratic has 2 factors, always. The zeros can be found using the quadratic formula and if there is a negative number under the radical(which was the problem when working in the real number system) this number can be converted into an imaginary number.

EX: SQRT(-a) = SQRT(a)*SQRT(-1)= i*SQRT(a) where a is a positive number.

Sample Problem: Factor x^2 + 4X + 7

Answer: (x+2-i*SQRT(3))*(x+2+i*SQRT(3))

Factual information:

- conjugates : for example conjugate of x+yi is x-yi
- magnitude : denoted by |z|, it is the distance between the complex # and 0 in the complex plane
- argument : written arg(z), is the measure of the angle in standard position z on the terminal side.
- x+yi : represents a complex number, the “x” is the *real part* of the complex number while the “y*i*” term is called the *imaginary part*.
- norm : the absolute value of a complex number is equal to the square root of its norm |z|= square root of N(z) (in other words the norm is $|z|^2$)
- multiplication law
- |zw| = |z| * |w|
- arg(zw) = arg(z) + arg(w)
- $i^2 = -1$

Review problems:

1. Graph the vector (5+2i)+(-8-8i) and label the endpoint z. Calculate the magnitude and norm of z. Find arg(z) in both degrees and in radians.

Answer: arrow drawn from the origin to point 5+2i and another arrow drawn from 5+2i to -3-6i.
|z|= 6.708 (rounded to three places) and N(z)= 45. arg z= 243.435º (rounded to three places) or 4.249 radians (rounded to three places)

2. How many imaginary and real roots does the function f(x)= (x+i)(x+2)(x+i) have?

Answer: 3

3. Find the conjugate, magnitude and argument of these complex numbers exactly, without using a calculator.

a. 2+2i b. 2-2i c. -2+2i d. -2-2i

Answers: a) 2-2i, 2 root 2 and 45º
b) 2+2i, 2 root 2 and 315º
c) -2-2i, 2 root 2 and 135º
d) -2+2i, 2 root 2 and 225º

4. Solve the following equation in the complex number system: “5x^2+6x+2=0”.

Answer: x= (-3/5)-(*i*/5) OR (-3/5)+(*i*/5)

5. Find the lowest degree polynomial with all real number coefficients with zeros: 4+7i, 3+2i.

Answer: x^4-14x^3+102x^2-302x-715