CP1 Math 4
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Systems of equations with infinitely many solutions, or with no solutions.

Example 1: Reduce the following augmented matrix to row-echelon form:

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & 1 & 4 & 8 \\ 1 & 2 & -1 & 1 \end{bmatrix}$$

Solution:

Step 1: Do $-2R_1 + R_2 \rightarrow R_2$ and $-R_1 + R_3 \rightarrow R_3$:

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -1 & 2 & 2 \\ 0 & 1 & -2 & 2 \end{bmatrix}$$

Step 2: Do $R_2 + R_1 \rightarrow R_1$ and $R_2 + R_3 \rightarrow R_3$:

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & -1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now we are stuck. This corresponds with the system of equations

$$\begin{cases} +x & +3z & = & 5 \\ -y & +2z & = & 2 \end{cases}$$

which can be written

$$\begin{cases} x & = & 5 - 3z \\ y & = & -2 + 2z \end{cases}$$

By taking different values of *z*, we can get an infinite number of solutions. In effect, these are parametric equations for *x* and *y* in terms of the parameter *z*.

Example 2: This is problem 5 on page 545.
Solve the system of equations using Gaussian Elimination.

$$\begin{cases} +x & +y & +z & = & -3 \\ 4x & -y & & = & -5 \\ -3x & +2y & +z & = & 4 \end{cases}$$

Solution:

The augmented matrix for this system is

$$\begin{bmatrix} 1 & 1 & 1 & -3 \\ 4 & -1 & 0 & -5 \\ -3 & 2 & 1 & 4 \end{bmatrix}$$

Step 1: $-4R_1 + R_2 \to R_2$ and $3R_1 + R_3 \to R_3$.

$$\begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & -5 & -4 & 7 \\ 0 & 5 & 4 & -5 \end{bmatrix}$$

Step 2: $R_2 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & -5 & -4 & 7 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

This corresponds with the system

$$\begin{cases} +x & +y & +z & = & -3 \\ & -5y & -4z & = & 7 \\ & & 0 & = & 2 \end{cases}$$

This system has no solution.