Sign Charts and more rational function graphs:

I. Sign charts: Class problem answer, and more problems to try.

II. Rational function graphs: Quiz problem 2 solution to discuss, and another problem to try.

Sign charts

Textbook references:

Examples: p. 236 Example 1, p. 239 Example 5

Homework:

Finish the problems on this handout.

Notes: Class problem solution

Problem: Make a sign chart for the following function that shows where the function is positive and where the function is negative.

\[ f(x) = \frac{(x + 3)(x - 4)(x^2 + 5)}{(x - 2)} \]

Solution:
The zeros of the numerator and denominator are \{-3, 2, 4\}. Use these to separate the number line into intervals. Analyze the function in each interval. Note that the \((x^2 + 5)\) is always positive. The other factors are positive in some intervals, negative in others.

\[
\begin{array}{cccc}
(-)(-)(+) & (+)(-)(+) & (+)(-)(+) & (+)(+)(+) \\
(-) & (-) & (+) & (+) \\
Negative & Positive & Negative & Positive
\end{array}
\]

More problems to try

1. Make a sign chart for the following function that shows where the function is positive and where the function is negative.

\[ f(x) = \frac{(x - 1)(x + 2)^2(x - 5)}{(x + 3)} \]
2. Make a sign chart for the following function that shows where the function is positive and where the function is negative.

\[ f(x) = \frac{(x + 3)|x + 1|}{(x - 5)^3} \]

3. Make a sign chart for the following function that shows where the function is positive and where the function is negative.

\[ f(x) = \frac{x\sqrt{x}}{2x - 3} \]
Rational function graphs

Solution to problem 2 from rational function quiz

Consider \( f(x) = \frac{7x^3 + 14x^2 + 7x}{2x^3 - 18} \).

a. Briefly explain how to algebraically find the horizontal asymptote of \( f(x) \), then write two limit statements describing the function near this asymptote (one describing the left end behavior, the other describing the right end behavior).

The leading terms \( \frac{7x^3}{2x^3} \) reduce to \( \frac{7}{2} \) so the horizontal asymptote is \( y = \frac{7}{2} \) or 3.5.

\[ \lim_{x \to \infty} f(x) = \frac{7}{2} \quad \text{and} \quad \lim_{x \to -\infty} f(x) = \frac{7}{2}. \]

b. Through factoring, this function be rewritten as \( f(x) = \frac{7(x+1)^2}{2(x+3)(x-3)} \). Find the \((x, y)\) coordinates of all removable discontinuities (holes) in the graph of \( f(x) \). Write a limit statement describing the function near each hole.

The only factors that can be reduced are the x’s, so there’s just one hole, at \( x = 0 \).

Evaluate the reduced function \( \frac{7(x+1)^2}{2(x+3)(x-3)} \) at \( x = 0 \) to get \( y = -\frac{7}{18} \).

So the hole is at \( (0, -\frac{7}{18}) \), and \( \lim_{x \to 0} f(x) = -\frac{7}{18}. \)
Another problem to try

Consider \( f(x) = \frac{2x^3 + 10x^2 + 4x - 16}{3x^3 - 6x^2 - 24x} \).

a. Briefly explain how to algebraically find the horizontal asymptote of \( f(x) \), then write two limit statements describing the function near this asymptote (one describing the left end behavior, the other describing the right end behavior).

b. Through factoring, this function be rewritten as \( f(x) = \frac{2(x + 2)(x - 1)(x + 4)}{3x(x + 2)(x - 4)} \).

Find the \((x, y)\) coordinates of all removable discontinuities (holes) in the graph of \( f(x) \). Write a limit statement describing the function near each hole.

c. [For this part, you may use either of the \( f(x) \) formulas above, and also you may consult a calculator graph if you wish.] For each vertical asymptote of \( f(x) \), write two limit statements describing the function graph on each side of the asymptote.