**Transformations extension and review**

*Objective:* Extend and review knowledge of transformations in preparation for a half-period test on transformations at our next class on Thursday 10/15 (H Block) or Friday 10/16 (G Block)

**Transformation rules**
You need to be fully knowledgeable of these transformation rules and prepared for an assessment either with or without a calculator.

### Vertical transformations

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Overall graphical change</th>
<th>How each point’s coordinates change</th>
<th>Function formula change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translations</td>
<td>shift up by $k$ units</td>
<td>add $k$ to every $y$-coordinate</td>
<td>$(\text{original function}) + k$</td>
</tr>
<tr>
<td></td>
<td>shift down by $k$ units</td>
<td>subtract $k$ from every $y$-coordinate</td>
<td>$(\text{original function}) - k$</td>
</tr>
<tr>
<td>Dilations</td>
<td>stretch by a factor of $n$, where $n &gt; 1$</td>
<td>multiply every $y$-coordinate by $n$</td>
<td>$n \cdot (\text{original function})$</td>
</tr>
<tr>
<td></td>
<td>shrink by a factor of $n$, where $0 &lt; n &lt; 1$</td>
<td>multiply every $y$-coordinate by $n$</td>
<td>$n \cdot (\text{original function})$</td>
</tr>
<tr>
<td>Reflection</td>
<td>flip across the $x$-axis</td>
<td>change every $y$-coordinate to opposite</td>
<td>$-(\text{original function})$</td>
</tr>
</tbody>
</table>

### Horizontal transformations

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Overall graphical change</th>
<th>How each point’s coordinates change</th>
<th>Function formula change /example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translations</td>
<td>shift right by $h$ units</td>
<td>add $h$ to every $x$-coordinate</td>
<td>replace $x$ with $(x - h)$</td>
</tr>
<tr>
<td></td>
<td>shift left by $h$ units</td>
<td>subtract $h$ from every $x$-coordinate</td>
<td>replace $x$ with $(x + h)$</td>
</tr>
<tr>
<td>Dilations</td>
<td>stretch by a factor of $n$, where $n &gt; 1$</td>
<td>Example: If you want to stretch a graph horizontally by a factor of 3: Multiply the $x$-coordinates of the points by 3. Leave the $y$-coordinates unchanged.</td>
<td>Example: if you want to stretch by a factor of 3: replace $x$ with $(\frac{x}{3})$</td>
</tr>
<tr>
<td></td>
<td>shrink by a factor of $n$, where $0 &lt; n &lt; 1$</td>
<td>Example: If you want to shrink a graph horizontally by a factor of 1/3: Divide the $x$-coordinates of the points by 3. Don’t change the $y$-coordinates.</td>
<td>Example: if you want to shrink by a factor of 1/3: replace $x$ with $(\frac{3x}{2})$</td>
</tr>
<tr>
<td>Reflection</td>
<td>flip across the $y$-axis</td>
<td>change every $x$-coordinate to opposite</td>
<td>replace $x$ with $(-x)$</td>
</tr>
</tbody>
</table>

### Problems

7. For each of the following, identify a transformation or sequence of transformations that transforms the graph of $f(x)$ into the graph of $g(x)$.

*Note:* In nearly all of our past problems like this, $f(x)$ has been one of the basic functions and $g(x)$ has been a modified version of that function. However, today’s questions will have more variety, such as $g(x)$ being the simpler function, or $f(x)$ and $g(x)$ both being modified versions of some basic function. You’ll need to think carefully about how to use your transformation knowledge in these situations.

- a. from $f(x) = \sqrt{x}$ to $g(x) = \sqrt{x} - 5$
- b. from $f(x) = \sqrt{x - 5}$ to $g(x) = \sqrt{x}$
- c. from $f(x) = \sqrt{x - 5}$ to $g(x) = \sqrt{x} + 4$
- d. from $f(x) = x^4 + x - 7$ to $g(x) = x^4 + x - 3$
7. (continued) Identify a transformation or sequence of transformations that transforms the graph of \(f(x)\) into the graph of \(g(x)\).

\[\begin{align*}
e. & \quad \text{from } f(x) = 3^x \text{ to } g(x) = 3^{\left(\frac{x}{4}\right)} \\
f. & \quad \text{from } f(x) = 3^{\left(\frac{x}{4}\right)} \text{ to } g(x) = 3^x \\
g. & \quad \text{from } f(x) = \cos(x) \text{ to } g(x) = \cos(\pi x) \\
h. & \quad \text{from } f(x) = \cos(\pi x) \text{ to } g(x) = \cos(x) \\
i. & \quad \text{from } f(x) = \cos(\pi x) \text{ to } g(x) = \cos(2x) \\
j. & \quad \text{from } f(x) = x^3 \text{ to } g(x) = 2 (x - 3)^3 - 1 \\
k. & \quad \text{from } f(x) = 2 (x - 3)^3 - 1 \text{ to } g(x) = x^3
\end{align*}\]

8. For each of the following, begin from \(f(x) = \frac{1-x}{\sqrt{x}}\), and write the function that results from the stated transformation(s). You do not have to simplify your answers.

\[\begin{align*}
a. & \quad \text{reflect across the } y\text{-axis} \\
b. & \quad \text{reflect across the } y\text{-axis and reflect across the } x\text{-axis} \\
c. & \quad \text{translate up by 1 and translate right by 5} \\
d. & \quad \text{horizontally shrink by a factor of } 1/4 \\
e. & \quad \text{horizontally stretch by a factor of } 4 \\
f. & \quad \text{vertically stretch by a factor of } 5, \text{ then translate left by } 0.3 \\
g. & \quad \text{translate down by 2, then vertically shrink by a factor of } 1/3, \text{ then reflect across the } x\text{-axis}
\end{align*}\]

9. Graphs of \(F(x)\) and \(G(x)\) are shown on the grid. The transformation from \(F(x)\) to \(G(x)\) can be regarded as either a vertical stretch or as a horizontal shrink.

\[\begin{align*}
a. & \quad \text{What is the vertical stretch factor from } F(x) \text{ to } G(x)\? \\
b. & \quad \text{What is the horizontal shrink factor from } F(x) \text{ to } G(x)\? \\
c. & \quad \text{The function formula for } F(x) \text{ is: } F(x) = \sqrt{x}. \text{ Find two possible function formulas for } G(x), \text{ one using the vertical stretch from part a, the other using the horizontal shrink from part b.}
\end{align*}\]
10. Let \( f(x) \) be the piecewise-linear function shown in the first graph below. Outside the edges of the grid, the graph of \( f(x) \) continues outward infinitely along the horizontal lines \( y = 1 \) and \( y = 0 \).

a. Find a sequence of four transformations that would transform the graph of \( y = f(x) \) into the graph of \( y = -3f(x+2)+1 \). Describe the transformations geometrically in the left column, and write the equation after each transformation in the right column.

<table>
<thead>
<tr>
<th>Beginning from…</th>
<th>( y = f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st transformation:</td>
<td>equation:</td>
</tr>
<tr>
<td>2nd transformation:</td>
<td>equation:</td>
</tr>
<tr>
<td>3rd transformation:</td>
<td>equation:</td>
</tr>
<tr>
<td>4th transformation:</td>
<td>( y = -3f(x+2)+1 )</td>
</tr>
</tbody>
</table>

b. Complete this sequence of graphs for the five equations listed above, starting with \( y = f(x) \) and ending with \( y = -3f(x+2)+1 \).
Answers

7. a. translate right by \(5\)
   b. translate left by \(5\) (because it needs the reverse of the transformation in part a)
   c. translate left by \(9\)
   d. translate up by \(4\)
   e. horizontal stretch by \(4\)
   f. horizontal shrink by \(\frac{1}{4}\) [or “by 4”] (reverse of part e)
   g. horizontal shrink by \(\frac{1}{\pi}\) [or “by \(\pi\”]\n   h. horizontal stretch by \(\pi\)
   i. horizontal stretch by \(\pi/2\) (or you might have used two steps: stretch by \(\pi\) then shrink by \(2\))
   j. One correct answer: translate right by \(3\), vertical stretch by \(2\), translate down by \(1\).
      There are other correct orders for these three transformations, but also some wrong orders.
      Specifically, the translation down has to come after the vertical stretch.
   k. Compared to part j, use reverse transformations in the reverse order.
      One correct answer: translate up by \(1\), vertical shrink by \(1/2\), translate left by \(3\).

8. a. \(\frac{1-(-x)}{\sqrt{-x}}\)    b. \(-\left(\frac{1-(-x)}{\sqrt{-x}}\right)\)    c. \(\frac{1-(x-5)}{\sqrt{x-5}}+1\)    d. \(\frac{1-4x}{\sqrt{4x}}\)
   e. \(\frac{1-\frac{x}{2}}{\sqrt{\frac{x}{4}}}\)    f. \(\left(\frac{1-(x+0.3)}{\sqrt{x+0.3}}\right)\)    g. \(-\frac{1}{3}\left(\frac{1-x}{\sqrt{x}}-2\right)\)

9. a. 2    b. \(\frac{1}{4}\) [or \(4\)]    c. \(G(x) = 2\sqrt{x}, \ G(x) = \sqrt{4x}\) (and note these answers are equal)

10. One possible answer is shown below. There are other correct orders that these transformations could be done, but also some incorrect orders. Your answer is correct if you have the correct final equation and a final graph as shown at the right.

   • translate left by \(2\)    \(y = f(x + 2)\)
   • vertical stretch by \(3\)    \(y = 3f(x + 2)\)
   • reflect across \(x\)-axis    \(y = -3f(x + 2)\)
   • translate up by \(1\)    \(y = -3f(x + 2) + 1\)