Modeling with sinusoidal functions: Word Problems

1. Man on a Waterwheel Problem  Perhaps you have seen the Tom Cruise movie, *The Last Samurai*. In one scene, a man is tied to a water wheel. Assume the wheel has a diameter of 10 m and the center of the wheel is 3 m above the water. When you first notice the man on the wheel (which moves counterclockwise), the man is at 3 o'clock. Four seconds later, he is at 12 o'clock. The question is, how long does he spend under water with each revolution of the wheel? (Hint: use your calculator to find intersections.)

2. Buried Treasure Problem  You seek a treasure that is buried in the side of a mountain. The mountain range has a sinusoidal cross section (see figure). The valley to the left is filled with water to a depth of 50 m, and the top of the range is 150 m above the water level. You set up an x-axis at water level and a y-axis 200 m to the right of the deepest part of the water. The top of the mountain is at $x = 400$ m.
   a. Write an equation expressing $y$ in terms of $x$ for points on the surface of the mountain.
   b. The treasure is located within the mountain at the point $(130, 40)$. Which would be a shorter way to dig to the treasure, a horizontal tunnel or a vertical tunnel? Justify your answer.

3. Sunspot Problem  For several hundred years, astronomers have kept track of the number of solar flares, or "sunspots," which occur on the surface of the sun. The number of sunspots counted in a given year varies periodically from a minimum of about 10 per year to a maximum of about 110 per year. Between the maximums that occurred in the years 1750 and 1948, there were 18 complete cycles.
   a. What is the period of the sunspot cycle? Sketch two sunspot cycles, starting in 1948.
   b. Write an equation expressing the number of sunspots per year in terms of the year.
   c. How many sunspots would you expect this year?
   d. What is the first year after 2000 in which the number of sunspots will be about 35? When will it be a maximum?

4. Sound Wave Problem  The hum you hear on a radio when it is not tuned to a station is a sound wave of 60 cycles per second.
   a. Is the 60 cycles per second the period or the frequency?
   b. If it is the period, find the frequency. If it is the frequency, find the period.
   c. The wavelength of a sound wave is defined to be the distance the wave travels in one period. If sound travels at 1100 ft/s, find the wavelength of a 60-cycle-per-second sound wave.
   d. The lowest musical note the human ear can hear is about 16 cycles per second. In order for the pipe on a church organ to generate this note, the pipe must be exactly half as long as the wavelength. How long an organ pipe would be required to generate a 16-cycle-per-second note?
Name: ____________________________  Honors Advanced Math
January 10, 2014  Solving with sinusoids

Solutions:
1. Waterwheel Problem
   \[ y = f(x) = 5\cos\left(\frac{2\pi}{16}(x - 4)\right) + 3 = 5\sin\left(\frac{2\pi}{16}(x)\right) + 3 \quad \text{time spent under water} = 4.723 \text{ seconds} \]

2. Buried Treasure Problem
   a) \[ y = f(x) = 100\cos\left(\frac{2\pi}{1200}(x - 400)\right) + 50 = 100\sin\left(\frac{2\pi}{1200}(x - 100)\right) + 50 \]
   b) \[ 40 = 100\sin\left(\frac{2\pi}{1200}(x - 100)\right) + 50 \Rightarrow x = 80.869 \quad \text{so horizontal hole is 49.13 m deep} \]
   c) \[ y = f(130) = 100\sin\left(\frac{2\pi}{1200}(30)\right) + 50 = 65.643 \quad \text{so vertical hole is 25.643 m deep} \quad \text{Dig Vertically} \]

3. Sunspots Problem
   a) period \( T \) = 11 years
   b) \[ s = f(y) = 50\cos\left(\frac{2\pi}{11}(y)\right) + 60 = 50\sin\left(\frac{2\pi}{11}(y + \frac{11}{4})\right) + 60 \]
   c) \[ s = f(2007) = 50\cos\left(\frac{2\pi}{11}(2007 - 1948)\right) + 60 = 27.257 \approx 27 \text{ sunspots} \]
   d) 35 sunspots in 2006, max in 2003

4. Sound Wave Problem
   a. frequency
   b. period = \( \frac{1}{\text{frequency}} = \frac{1 \text{ secs}}{60 \text{ wave}} \)
   c. wavelength \( \lambda = \frac{1100 \text{ ft}}{60 \text{ waves}} \cdot \frac{1 \text{ sec}}{60 \text{ waves}} = 18.3 \text{ ft} \)
   
   \[ \text{pipelength} = \frac{\lambda}{2} = \frac{1100 \text{ ft}}{16 \text{ waves}} \cdot \frac{1 \text{ sec}}{16 \text{ waves}} \cdot \frac{1}{2} = 34.375 \text{ ft} = 34' \frac{1}{2} \text{ ft} \]