Students should be able to:

- Understand and use geometric vocabulary and geometric symbols (\(\cong\), \(\perp\), \(\parallel\), etc)
- Write proofs using accurate definitions, theorems, postulates, and properties of congruence (reflexive property, transitive property, addition property, etc)
- Identify congruent triangles, including those in complex diagrams, and justify that two triangles are congruent using 1 of 5 methods: SSS, SAS, ASA, AAS, HL
- Understand that all corresponding parts of congruent triangles are congruent, and use that fact (“CPCTC”) to show specific corresponding segments/angles of pairs of congruent triangles must be congruent
- Complete multi-step proofs that require you to go several steps beyond CPCTC, prove multiple pairs of triangles congruent, and/or use segment/angle addition and the addition property
- Know the difference between a median and an altitude
- Understand and apply the Perpendicular Bisector Theorem, Isosceles Triangle Theorem, and their converses; know the Vertical Angles Theorem as well as the Exterior Angle Theorem
- Understand that not all lines cut by a transversal are parallel
  - Know various angle relationships you can prove to be true when you KNOW the lines are parallel
  - Know various methods of PROVING the lines are parallel, based on given angle relationships

Vocabulary

<table>
<thead>
<tr>
<th>Acute Angle</th>
<th>Consecutive Angles</th>
<th>Isosceles Triangle</th>
<th>Right Angle</th>
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</thead>
<tbody>
<tr>
<td>Acute Triangle</td>
<td>Diagonal</td>
<td>Median</td>
<td>Right Triangle</td>
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<tr>
<td>Adjacent Angles</td>
<td>Equiangular</td>
<td>Midpoint</td>
<td>Scalene Triangle</td>
</tr>
<tr>
<td>Altitude</td>
<td>Equilateral</td>
<td>Obtuse Angle</td>
<td>Straight Angle</td>
</tr>
<tr>
<td>Angle Bisector</td>
<td>Concurrent</td>
<td>Obtuse Triangle</td>
<td>Straight Angle</td>
</tr>
<tr>
<td>Bisect</td>
<td>Exterior Angle of a Polygon</td>
<td>Parallel</td>
<td>Supplementary Angles</td>
</tr>
<tr>
<td>Trisect</td>
<td>Interior Angle of a Polygon</td>
<td>Perpendicular</td>
<td>Complementary Angles</td>
</tr>
<tr>
<td>Conjecture</td>
<td>Invariant</td>
<td>Perpendicular Bisector</td>
<td>Vertical Angles</td>
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Theorems, Postulates, and Properties for Use in Proofs

Triangle Congruency Methods:

<table>
<thead>
<tr>
<th>SAS postulate</th>
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<tr>
<td>Given two triangles, if two corresponding sides and the corresponding angle between them are congruent, then the triangles are congruent.</td>
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<table>
<thead>
<tr>
<th>SSS postulate</th>
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<tr>
<td>Given two triangles, if the three corresponding sides are congruent, then the triangles are congruent.</td>
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<th>ASA postulate</th>
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### HL Congruence
If two right triangles have congruent corresponding hypotenuses and a pair of congruent corresponding legs, then the triangles are congruent.

(Note: In a proof, you need to show the congruent hypotenuse, congruent leg, and presence of a right angle, but you do not need to show the right angles are congruent.)

### AAS Congruence
Given two triangles, if two corresponding angles and a corresponding side not between them are congruent, then the triangles are congruent.

### Postulates:

#### Linear Pair Postulate
If two angles form a linear pair, then they are supplementary.

In a proof: To say $\angle 1 + \angle 2 = 180^\circ$, your reason can be “Linear pair postulate,” “linear pairs are supplementary,” or “they form a straight angle.”

#### Segment Addition Postulate (whole = sum of parts)
If a point $B$ lies on segment $\overline{AC}$, then $\overline{AC} = \overline{AB} + \overline{BC}$.

#### Angle Addition Postulate (whole = sum of parts)
If a point $D$ lies on the interior of $\angle ABC$, then $\angle ABC = \angle ABD + \angle DBC$.

#### Parallel postulate.
Given a line and a point not on the line there is exactly one line passing through the point that is parallel to the line.

(We used this postulate to prove that a triangle's interior angles add to $180^\circ$)

### Theorems:

#### Vertical Angles Theorem (VAT)
Vertical angles are congruent.

#### Triangle Angle Sum Theorem.
The sum of the angles in a triangle is $180^\circ$

#### Triangle Exterior Angle Theorem
The measure of an exterior angle of a triangle is equal to the sum of the remote interior angles.
| **Isosceles Triangle Theorem and its Converse** | **Theorem:** If a triangle has two congruent sides, then the angles opposite those sides are also congruent. |
| **Converse:** If a triangle has two congruent angles, then the sides opposite those angles are also congruent. |

| **Applying ITT and its converse to Equilateral Triangles:** | If a triangle is equilateral, then it is equiangular. |
| If a triangle is equiangular, then it is equilateral. |

| **Perpendicular Bisector Theorem and its Converse** | **Theorem:** If a point is on the perpendicular bisector of a segment, then it is equidistant from the two endpoints of the segment. (In a proof: You know a line is the perpendicular bisector of a segment, you use PBT to prove a point on that line is equidistant from the endpoints of the segment) |
| **Converse:** If a point is equidistant from the two endpoints of the segment, then it is on the perpendicular bisector of that segment (In a proof: You know a point is equidistant from the endpoints of a segment; you use PBT converse to say that point is on the perpendicular bisector of the segment. Repeat with a second point—remember you need two points to determine a line). |

| **Postulates & Theorems about Parallel Lines:** |
| **PAI Postulate (P→AI)** | If lines are Parallel, then Alternate Interior angles are congruent. |
| **Alternate Exterior Angle Theorem** | If lines are parallel, then alternate exterior angles are congruent. |
| **Corresponding Angle Theorem** | If lines are parallel, then corresponding angles are congruent. |
| **Consecutive Angles Theorem (Same-Side Interior Angles Theorem)** | If lines are parallel, then consecutive angles are supplementary. |
| **Same-Side Exterior Angles Theorem** | If lines are parallel, then same-side exterior angles are supplementary. |
AIP Postulate (Al→P)

If Alternate Interior angles are congruent, then lines are Parallel.

You can use AIP to prove the converses of the other angle relationships listed above.

Properties Commonly Used in Proofs:

Reflexive Property:

Any segment is congruent to itself. Any angle is congruent to itself.

Transitive Property:

If two angles/segments are congruent to the same angle/segment, then they are congruent to each other.

Applications:

- Supplements of congruent angles are congruent
- Complements of congruent angles are congruent

Addition Property:

If a segment/angle is added to congruent segments/angles, then the sums are congruent.

Proof Writing Tips

One of the best ways of figuring out how to write a proof is to work backwards from the conclusion. Ask yourself the following questions:

1) What am I trying to prove?
   - Congruent triangles? Congruent segments? Congruent angles?
   - Another relationship? (Isosceles triangles, midpoints, bisectors, etc.)

2) What are the possible methods of proving that?
   - Congruent triangles - SSS, SAS, ASA, AAS
   - Congruent segments - CPCTC
   - Congruent angles - CPCTC
   - Another relationship - fulfill the needs of a definition or a theorem

3) Do I have enough information to prove two triangles are congruent?
   - Label the figure with known information (given as well as inferred).
   - Do the triangles share a side or angle so that it is congruent in both triangles?

4) Does every step serve a purpose and a valid reason?
   - Do you have steps that don’t lead anywhere (except the conclusion)? Do all of your steps follow a logical order?
**Practice:** Complete the following proofs on a separate sheet of paper.

1. **Given:** \( XR \parallel SY \)
   \( RS \) bisects \( XY \)

   **Prove:** \( \Delta RMX \cong \Delta SMY \)

2. **Given:** \( DC \parallel AB \)
   \( DC \cong AB \)

   **Prove:** \( AD \parallel BC \)

3. **Given:** \( \angle 5 \cong \angle 6 \)
   \( AT \cong HT \)

   **Prove:** \( MT \) bisects \( \angle AMH \)
4. **Given**: \( \angle 1 \cong \angle 2 \)
\( \triangle BUG \) is isosceles with base \( \overline{BG} \)

**Prove**: \( UA \perp BG \)

YOU MAY NOT USE REFLEXIVE SIDE \( UA \) IN YOUR PROOF!!

5. **Given**: \( AB \cong CD; AD \cong BC \)
\( AX \perp BD; CY \perp BD \)

**Prove**: \( AX \cong CY \)

6. **Given**: \( \angle 1 \cong \angle 2, \angle 3 \cong \angle 4 \)
Points \( B, E, \) and \( D \) are collinear

**Prove**: \( BA \cong BC \)
7. A student built this GeoGebra diagram, in which $\overline{AD} \parallel \overline{CE}$, $\overline{AB}$ bisects $\angle DAC$, and $\overline{CB}$ bisects $\angle ECA$. She found that when she moved the points around, the measure of $\angle B$ was invariant.

What is the measure of $\angle B$?
Prove your answer.

8. Draw a diagram to represent the given statement. Then determine the given(s) and what you need to prove, and write the proof.

If an altitude is drawn to the base of an isosceles triangle, then it is also a median.

Given: ___________________________

Labeled Diagram:

Prove: ___________________________

9. Given: $\triangle ABC \cong \triangle ADC$

Prove: $\overline{AC}$ is the $\perp$ bisector of $\overline{BD}$
10. **Given:** $ME \cong SE$, $MH \cong SH$

Points $M$, $H$, and $F$ are collinear

Points $S$, $H$, and $K$ are collinear

**Prove:** $KH \cong FH$

11. **Given:** $QM \perp MP$ and $QS \perp SP$

$MP \cong PS$

**Prove:** $PN$ bisects $\angle MNS$

12. **Given:** $MP$ is the $\perp$ bisector of $LN$

Points $M$, $Q$, $R$, and $P$ are collinear

**Prove:** $\angle QLP \cong \angle QNP$